University of California, Berkeley
Physics 110B, Spring 2004 (*Strovink*)

HINTS FOR ASSIGNED EXERCISES 60-72

60. Interference of two beams following different paths.

Consider two beams A and B. At (early) plane P, the relative properties of the two beams are well understood; for example, a single laser beam may be split into two. Between plane P and (late) plane Q, the beams follow different paths A and B through a nondispersive medium ($v_{\rm group} = v_{\rm phase}$); by the time they reach plane Q they have recombined. (For example, a Michelson interferometer may be interposed between the two planes.) At P and Q define

$$\text{physical } \vec E_{A,B}(P,Q) \equiv \mathrm{Re}\big(\vec E^{A,B}_{P,Q}\, e^{-i\omega t}\big)$$

(this is four equations). On the left-hand side are physical fields that vary rapidly ($\approx$ sinusoidally) with time t; on the right-hand side are complex fields $\vec E^{A,B}_{P,Q}$ having magnitudes that are fixed, but phases that vary more slowly, over many sinusoidal periods. This slow variation may occur separately for the x and y components of a beam's electric field – in which case the beam is completely unpolarized – or it may occur in lockstep for the x and y components together, in which case the beam remains fully polarized.

The optical phase shifts for paths A and B are equal to

$$\omega\tau_{A,B} \equiv \int_P^Q \vec k_{A,B}\cdot d\vec r_{A,B}$$
$$\tau \equiv \tau_B - \tau_A \ ,$$

where $\vec k_{A,B}(\vec r)$ is the wave vector for beam A or B, respectively, and $d\vec r_{A,B}$ lies along the path for beam A or B.

(a.)
The (undispersed) physical waves remain functions of $(\vec k_{A,B}\cdot\vec r_{A,B} - \omega t)$, even as these slow phase variations occur. Use this fact to show that

$$\text{physical } \vec E_{A,B}(Q)(t+\tau_{A,B}) =$$
$$\text{physical } \vec E_{A,B}(P)(t) \ .$$

Hint:
Temporarily assume that the complex fields $\vec E^{A,B}_{P,Q}$ are constant. Then show that the physical fields at P and Q, which are functions of $(\vec k_{A,B}\cdot\vec r_{A,B} - \omega t)$, are related by

$$\text{physical } \vec E_{A,B}(Q)(t') = \text{physical } \vec E_{A,B}(P)(t) \times$$
$$\times \exp\Big(i\big(\int_P^Q \vec k_{A,B}\cdot d\vec r_{A,B} - \omega(t'-t)\big)\Big) \ .$$

Now, allow the phases of $\vec E^{A,B}_{P,Q}$ to vary slowly. In the above equation, plug in

$$t' = t + \frac{1}{\omega}\int_P^Q \vec k_{A,B}\cdot d\vec r_{A,B}$$
$$\equiv t + \tau_{A,B} \ ,$$

so that the epoch of slow phase variation is the same at Q as it is at P.

(b.)
Using the result of part (a.), show that

$$\vec E^{A,B}_Q(t+\tau_{A,B}) = \vec E^{A,B}_P(t)\exp\left(i\omega\tau_{A,B}\right) \ .$$

Hint:
Express the physical fields in terms of the complex fields.

(c.)
At any other time t', the result of (b.) also holds. Choose $t' = t-\tau$. Show that

$$\vec E^B_Q(t+\tau_A) = \vec E^B_P(t-\tau)\exp\left(i\omega\tau_B\right) \ .$$

(d.)
The irradiance

$$I = \tfrac{1}{2}\sqrt{\tfrac{\epsilon}{\mu}}\,|\vec E^A + \vec E^B|^2$$

for the superposition of the two beams satisfies

$$2\sqrt{\tfrac{\mu}{\epsilon}}\,I_{P,Q} = |\vec E^A_{P,Q}|^2 + |\vec E^B_{P,Q}|^2 + 2\mathrm{Re}\big(\vec E^{A*}_{P,Q}\cdot\vec E^B_{P,Q}\big)$$

Using the results of (b.) and (c.), show that

$$I_Q(t+\tau_A) = I^A + I^B + \\ + \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\, 2\,\mathrm{Re}\big(\vec{E}_P^{A*}(t)\cdot\vec{E}_P^{B}(t-\tau)\exp(i\omega\tau)\big)\ ,$$

where $I^{A,B}$ are the (time-independent and space-independent) single-beam irradiances.

Hint:

In the supplied expression for $I_Q(t+\tau_A)$, express the $\vec{E}_Q^{A,B}$ in terms of the $\vec{E}_P^{A,B}$.

(e.)

Taking a long-time average (long compared to the characteristic time over which the complex electric field phases vary), obtain as a final step the *master equation for two-beam interference:*

$$\langle I_Q^{A+B}\rangle(\tau) = I^A + I^B + \\ + \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,\langle 2\,\mathrm{Re}\big(\vec{E}_P^{A*}(t)\cdot\vec{E}_P^{B}(t-\tau)\exp(i\omega\tau)\big)\rangle\ ,$$

where $\langle\rangle$ denotes a long-time average, and I_Q^{A+B} is the combined irradiance at plane Q.

Hint:

Note that $\langle I_Q^{A+B}\rangle(\tau)$ is not the combined irradiance at time τ. First, τ is not a time – it is the difference of the optical phase shifts for paths B and A. Second, $\langle I_Q^{A+B}\rangle$ can't be a function of time, because it is already a long-term time average. Rather, $\langle I_Q^{A+B}\rangle(\tau)$ is a time-independent irradiance that depends on the path difference that is parametrized by τ.

61.

Please refer to the notation and results of the previous problem. Define the *correlation* $\Gamma^{AB}(\tau)$ as

$$\Gamma^{AB}(\tau) \equiv \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}}\,\langle\vec{E}_P^{A*}(t)\cdot\vec{E}_P^{B}(t-\tau)\exp(i\omega\tau)\rangle\ ,$$

and define the *degree of partial coherence* $\gamma^{AB}(\tau)$ as

$$\gamma^{AB}(\tau) \equiv \frac{\Gamma^{AB}(\tau)}{\sqrt{I^A I^B}}\ .$$

(a.)

Show that the result of the last part of the previous problem can be written

$$\langle I_Q^{A+B}\rangle(\tau) = I^A + I^B + 2\sqrt{I^A I^B}\,\mathrm{Re}\,\gamma^{AB}(\tau)\ .$$

(b.)

If the screen Q in a two-beam interference setup deviates slightly from perfect perpendicularity to the beams, deviations of order 10-100 occur in $\omega\tau$ across the screen. For most sources these deviations do not cause a significant change in $\vec{E}_P^{B}(t-\tau)$, but they do cause the phase of $\exp(i\omega\tau)$ to change dramatically. Correspondingly there appear on the screen many light and dark bands ("fringes"), at the center of which the respective irradiances are $I_{\max}$ and $I_{\min}$. Define the *fringe visibility* $\mathcal{V}$ as

$$\mathcal{V} \equiv \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}\ .$$

If $I^A = I^B$, show that

$$\mathcal{V} = |\gamma^{AB}(\tau)|\ .$$

Hint:

In the middle of a bright (dark) fringe, $\gamma^{AB} = +|\gamma^{AB}|$ ($\gamma^{AB} = -|\gamma^{AB}|$).

(c.)

As an experimentalist, suppose that you are required to analyze the extent to which a mystery beam is polarized.

A standard approach would be to measure the elements of its Stokes vector (by observing the reduction in irradiance caused by four different optical devices – see Problem 59); knowing the Stokes vector, you could calculate the degree of polarization V (Problem 59(b.)).

Instead you decide to send the beam into a Michelson interferometer with two exactly equal-length paths A and B. Observing the resulting fringe pattern on screen Q, you measure the fringe visibility $\mathcal{V}$ (as defined in part (b.) of this problem).

Do you obtain any useful information about V by measuring $\mathcal{V}$? If so, what is the relationship between the two?

Hint:

If there are slow random phase variations of the complex field $\tilde{E}_y$ with respect to $\tilde{E}_x$, a beam is completely unpolarized. If such phase variations were to begin, while the Michelson path lengths remained exactly equal, how would the observed fringe pattern change? That is, how would the interference of $\tilde{E}_x^A$ with $\tilde{E}_x^B$, and $\tilde{E}_y^A$ with $\tilde{E}_y^B$ be affected?

62.

We wish to use the light of Betelgeuse (angular diameter 0.047 arc second), passed through a 600 nm filter, as the source for a double-thin-slit Young's interference experiment.

(a.)

Assuming an adequately narrow filter bandpass, roughly estimate the maximum slit separation (in m) that would yield an interference pattern which isn't too badly washed out, *i.e.* with a fringe visibility $\mathcal{V}$ of order $\frac{1}{2}$.

Hint:

This part is a *transverse* coherence problem, involving a source rendered monochromatic by the narrow filter. In the paraxial approximation, first consider the classical two-thin-slit irradiance pattern $I(\psi) \propto \cos^2 \frac{kd}{2}\psi$ that would occur if Betelgeuse were a point source. Now allow the two (point) halves of Betelgeuse to separate. This causes the two halves of the classical irradiance pattern likewise to move apart. As they do so, the primary irradiance maximum (at $\psi = 0$) and first minimum (at $\psi = \pi/kd$) remain at the same positions, but become less extreme. Parametrize this separation by a phase $\pm\delta$ that is added to the argument of $\cos^2$. Solve for δ such that $\mathcal{V} = \frac{1}{2}$; then relate 2δ to the (supplied) angular separation of Betelgeuse's two halves. This fixes the slit separation d.

(b.)

Assuming an adequately small slit separation, roughly estimate the maximum filter bandpass (in nm) that would allow us to observe at least 20 fringes. With this choice of bandpass, what is the coherence length of the transmitted light?

Hint:

This part is a *temporal* coherence problem, involving a slit separation so small that, with a monochromatic source, a very large number of fringes would be visible. Call the central (brightest) fringe the 0^{th} fringe maximum. About 10 fringes to either side, the pattern is mostly washed out because the source is polychromatic. This occurs when the 10th fringe *maximum* of light with wavelength $\lambda_0 - \Delta\lambda/2$ coincides with the 10^{th} fringe *minimum* of light with wavelength $\lambda_0 + \Delta\lambda/2$. Use this fact to relate $\Delta\lambda$ to λ (see the related discussion in P×2 section 11-2). The coherence length of the transmitted light follows from P×2 Eq. (12-18).

63.

A monochromatic beam traveling in medium "0" is normally incident upon a substrate *"T"*. A single film "1" is interposed between the two media. The refractive indices are, respectively, n_0, n_1, and n_T. You may assume that all materials have the same magnetic permeability.

(a.)

Show that a film of thickness $\lambda_1/4$ (where λ_1 is the wavelength of light in the material i of which the film is made) will reduce the reflectance of the substrate to zero, provided that $n_1 = \sqrt{n_0 n_T}$.

Hint:

Remember that $\lambda_1 = \lambda_{\text{vacuum}}/n_1$. This allows the film's phase advance δ (*e.g.* in P×2 Eq. (19-24)) to take the simple value $\pi/2$. Substitute the resulting transfer matrix elements in the standard reflection formula, *e.g.* P×2 Eq. (19-36).

(b.)

Prove that interposing a single film of thickness $\lambda_1/4$ will *always* reduce the reflectance of the substrate, provided that $n_0 < n_1 < n_T$.

Hint:

After obtaining the amplitude reflectance $r_{0,\text{b}}$ for the cases (no film) and (b), form the ratio r_0^2/r_b^2 and (tentatively) set it > 1. Applying brute force, multiply through by all denominators and make many cancellations. Distill the resulting inequality into one that is obviously true, given that $n_0 < n_1 < n_T$. To check your algebra, keep in mind that every inequality should become an equality if n_1 is set to n_0 or n_1 is set to n_T (as these cases correspond to no film at all).

64.

Referring to the conditions of the previous problem, consider next the case of three films ("1", "2", and "3") interposed between the two media, such that film 1 adjoins medium 0 and film 3 adjoins medium T. Again, assume that all ma-

terials have the same magnetic permeability.
(a.)
Suppose that each film has thickness $\lambda_i/4$ (where λ_i is the wavelength of the beam in the particular material of which that film is made). Show that the reflectance of the substrate is reduced to zero when

$$\frac{n_1 n_3}{n_2} = \sqrt{n_0 n_T} \ .$$

Hint:
After you take the product of the transfer matrices, the result should have diagonal elements that vanish.
(b.)
An advantage of using three films instead of one (as in the previous problem) is that the band of wavelengths over which the reflectance is heavily suppressed can be made much broader. (Your expensive eyeglasses are coated with at least two films.) According to Pedrotti×2 Fig. 19-7, this benefit may be enhanced further if the middle film (2) is doubled in thickness from $\lambda_2/4$ to $\lambda_2/2$. In this case, what condition on n_0, n_1, n_2, n_3, and n_T reduces the reflectance to zero?
Hint:
Here the product of transfer matrices should be diagonal.

65.
Consider a high-reflectance stack of the type depicted in Pedrotti×2 Fig. 19-8. For specificity, assume that the stack consists of six double layers of MGF_2 ($n = 1.38$) and ZnS ($n = 2.35$). For simplicity, assume that the medium from which the light enters the stack (medium 0) and the medium into which the light exits the stack (medium T) are vacuum. Again, assume that all materials have the same magnetic permeability.
(a.)
Numerically, what fraction T of the incident irradiance is transmitted by the stack?
Hint:
Consider one double layer. When both films in the double layer have thickness $\lambda_i/4$ (where λ_i is the wavelength in the medium of layer i), the product of the transfer matrices for the two layers is diagonal. Therefore the transfer matrix for N double layers is simply the transfer matrix for one layer raised to the N^{th} power (see P×2 Eq. (19-50)).
(b.)
The stack is now modified as follows: the upstreammost three double layers are flipped around so that the stack indices are L(ow) H(igh) L H L H H L H L H L. This is a *Fabry-Perot interference filter*. It has a transmission *maximum* at the wavelength for which it was designed, as opposed to the transmission *minimum* achieved by the configuration of part (a.). Calculate the fraction T of the incident irradiance that is transmitted by the modified stack.
Hint:
When the order of the films is inverted, the diagonal elements of the transfer matrix for the double layer are interchanged. To what extremely simple form does the N-element transfer matrix reduce?

66. Green's theorem.
Denote by $\vec{G}$ a vector field, and start from the *divergence theorem*

$$\int \nabla \cdot \vec{G}\, d\tau = \oint \vec{G} \cdot \hat{n}\, da \ ,$$

where $\hat{n}$ is the (outward) direction of the surface area element $d\vec{a}$, and the left-hand integral extends over the volume enclosed by the right-hand surface.
(a.)
Substituting $\vec{G} = V\nabla U$, where V and U are scalar fields, show that

$$\int \bigl(\nabla V \cdot \nabla U + V\nabla^2 U\bigr)\, d\tau = \oint V \frac{\partial U}{\partial n}\, da \ .$$

Hint:
Use the fact that

$$(\nabla U) \cdot \hat{n} \equiv \frac{\partial U}{\partial n} \ .$$

(b.)
Show that

$$\int \bigl(V\nabla^2 U - U\nabla^2 V\bigr)\, d\tau = \oint \Bigl(V\frac{\partial U}{\partial n} - U\frac{\partial V}{\partial n}\Bigr)\, da \ .$$

Hint:
Repeat part (a.) with $\vec{G} = U\nabla V$. Then subtract one of the two resulting equations from the other.
(c.)
If V and U both satisfy the scalar Helmholtz equation,

$$\big(\nabla^2 + k^2\big)(U, V) = 0 \ ,$$

where k is a constant, show that

$$0 = \oint \Big(V \frac{\partial U}{\partial n} - U \frac{\partial V}{\partial n} \Big) \, da \ .$$

This is *Green's theorem for solutions to the scalar Helmholtz equation*.
Hint:
Substitute $k^2 U, V$ for $\nabla^2 U, V$ in the result of part (b.).

67. Fresnel-Kirchoff integral theorem.
Please use the notation and results of the previous problem.
(a.)
Consider a closed surface consisting of an inner sphere of radius R, centered at the origin, and an arbitrary closed outer surface $\mathcal{A}$. Apply the result of part (c.) to the combined surface. Take V to be an inward-propagating spherical wave

$$V = V_0 \frac{e^{i(kr+\omega t)}}{r} \ .$$

In the limit $R \to 0$, show that

$$U(0) = \tfrac{1}{4\pi} \oint \Big(\frac{e^{ikr}}{r} \frac{\partial U}{\partial n} - U \frac{\partial}{\partial n} \big(\frac{e^{ikr}}{r} \big) \Big) \, da \ ,$$

where the integral is taken only over $\mathcal{A}$. This is the *Kirchoff integral theorem*.
Hint:
Factoring out $V_0 \, e^{i\omega t}$, this is equivalent to showing that, in the limit $R \to 0$, the integral over the inner spherical surface reduces to $-4\pi \, U(0)$. (Note that the normal to this surface is $-\hat{r}$.)
(b.)
Now punch a hole ("aperture") in $\mathcal{A}$. Place a point source S outside $\mathcal{A}$; the origin (now called "observation point P") still lies inside $\mathcal{A}$. The source radiates an outward-propagating scalar spherical wave

$$U = U_0 \frac{e^{i(kr'-\omega t)}}{r'} \ ,$$

where $\vec{r}\,'$ is a vector from S to a point in space. Using the result of (a.), assume that the opacity of the remainder of $\mathcal{A}$ allows the integral to be carried out over only the aperture ("ap"). In the far zone limit $kr', kr \gg 1$, show that

$$U_P = \frac{-ikU_0 e^{-i\omega t}}{4\pi} \int_{\text{ap}} \frac{e^{ik(r+r')}}{rr'} \big(\hat{r} \cdot \hat{n} - \hat{r}' \cdot \hat{n} \big) \, da \ ,$$

where $\vec{r}$ ($\vec{r}\,'$) is a vector from P (point S) to a point on the element of aperture da, and $\hat{n}$ is the (outward from P) normal to da. This is the *Fresnel-Kirchoff integral theorem*; it is the starting point for the study of diffraction in the *scalar field approximation*.
Hint:
Substitute

$$U = U_0 \frac{e^{i(kr'-\omega t)}}{r'}$$

in the result of part (a.) and perform the indicated differentiation. In analogy to the hint to part (a.) of the previous problem, note that

$$\frac{\partial}{\partial n} \frac{e^{ikr'}}{r'} = \big(\nabla' \frac{e^{ikr'}}{r'} \big) \cdot \hat{n} \ ,$$

where ∇' is the gradient with respect to the coordinate $\vec{r}\,'$.

68. Knife-edge diffraction.
A plane wave of initial irradiance I_0 propagating along $\hat{z}$ is incident upon a semi-infinite totally absorbing screen lying in the $z = 0$ plane. The screen extends from $-\infty < x < \infty$ and $-\infty < y < 0$. An observer stationed at $(0, 0, z)$, where $kz \gg 1$, detects an irradiance I'. What is I'/I_0, and why?
Hint:
Using $z = 0$ as the aperture plane, equate the optical disturbance U_P to a Fresnel-Kirchoff integral over that plane, (a.) for the case in which there is no absorber, and (b.) for the knife-edge case in which the absorber covers $y < 0$. Considering that the source is a plane wave with no

y dependence, and that the observer is stationed at the up-down symmetry point $y = 0$, how is integral (b.) related to integral (a.)? How is U related to the irradiance?

69. Fourier diffraction.
The *convolution* of two functions $f(x)$ and $g(x)$, denoted by $(f \otimes g)(x)$, is defined by

$$f \otimes g \equiv \int_{-\infty}^{\infty} dx' \, f(x') \, g(x - x') \ .$$

Define the *Fourier transform* $\mathcal{F}_\mu\big(g(x)\big)$ by

$$\mathcal{F}_\mu\big(g(x)\big) \equiv \int_{-\infty}^{\infty} dx \, g(x) \, e^{-i\mu x} \ .$$

(a.)
As a warmup, prove that

$$f \otimes g = g \otimes f \ .$$

Hint:
Substitute $u' = x - x'$ in the convolution integral.
(b.)
For use in part (d.), prove that

$$\mathcal{F}_\mu\big(f(x) \otimes g(x)\big) = \mathcal{F}_\mu\big(f(x)\big) \, \mathcal{F}_\mu\big(g(x)\big) \ .$$

Hint:
Write $\mathcal{F}_\mu\big(f(x) \otimes g(x)\big)$ as a double integral $\int dx \int dx' \dots$. Reverse the order of integration and substitute $u = x - x'$. Express the result as the product of an integral over u of purely u-dependent terms, $\times$ an integral over x' of purely x'-dependent terms.
(c.)
If $f(x)$ is the aperture function for a pair of thin slits separated by d,

$$f(x) \propto \delta(x - \tfrac{d}{2}) + \delta(x + \tfrac{d}{2}) \ ,$$

and if $g(x)$ is the aperture function of a single slit of thickness a,

$$g(x) \propto \theta(x + \tfrac{a}{2}) - \theta(x - \tfrac{a}{2}) \ ,$$

show that $f \otimes g$ is the aperture function corresponding to two slits of thickness a, separated (centerline-to-centerline) by d.
Hint:
As your intuition develops, this proposition will become obvious, but here you are asked to show it formally. Do so by carrying out the convolution integral, evaluating the integrand at the points where one of the δ-functions is nonzero. Note that the θ-function $\theta(u)$ steps from 0 at $u < 0$ to 1 at $u > 0$.
(d.)
In the Fraunhofer approximation, where $\vec{r}\,'$ and $\vec{r}$ (cf. Problem 67) are paraxial and the wavefront curvature across the aperture is negligible, the scalar "optical disturbance" amplitude is

$$U_P(\mu, \nu) \propto \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \, g(x, y) e^{-i(\mu x + \nu y)} \ ,$$

where U_P is measured at the transform plane (X, Y), the aperture function g is measured at the aperture plane (x, y), μ and ν are defined by

$$\mu \equiv \frac{kX}{f} \quad \nu \equiv \frac{kY}{f} \ ,$$

and f is the focal length of the thin field lens located an equidistance f from the aperture and transform planes. Write down the diffraction pattern

$$\frac{I_N(\psi_x, \psi_y)}{I_1(0,0)}$$

for N slits of center-to-center separation $\Delta x = d$ and thicknesses $\delta x = a$ and $\delta y = b$, where

$$(\sin)\psi_x \equiv \frac{X}{f}$$
$$(\sin)\psi_y \equiv \frac{Y}{f} \ .$$

You may use the fact – directly obtainable by applying the Fourier transform – that

$$\frac{I_N(\psi_x)}{I_1(0)} = N^2 \frac{\sin^2\left(\frac{Nkd}{2} \sin \psi_x\right)}{\big(N \sin\left(\frac{kd}{2} \sin \psi_x\right)\big)^2}$$

for N thin slits of infinite length and separation d, and that

$$\frac{I(\psi_x)}{I(0)} = \mathrm{sinc}^2\left(\tfrac{ka}{2} \sin \psi_x\right)$$

for a single slit of infinite length and thickness a.
Hint:
Building on the result of part (c.), use your intuition to express the aperture function for N thick slits as the convolution of the aperture function for N thin slits and the aperture function for one thick slit. Also use the fact that the Fourier transform of the product of two functions of different variables (x and y here) is the product of the Fourier transforms. Then exploit the result of part (b.).

70. Quadruple slit.
Consider four equally spaced long ($\Delta y = \infty$) thin slits, located at $x = \pm\frac{d}{2}$ and $x = \pm\frac{3d}{2}$. As usual, $\tan\psi_x = \frac{dx}{dz}$ of the outgoing wavefront.
(a.)
Write down the standard result

$$\mathcal{R}(\psi_x) \equiv \frac{I(\psi_x)}{I(\psi_x = 0)}$$

for the Fraunhofer diffraction pattern from $N = 4$ equally spaced thin slits.
Hint:
See part (d.) of the previous problem.
(b.)
Consider the full diffracted *amplitude* to be the superposition of the diffracted *amplitudes* from a pair of slits at $x = \pm\frac{d}{2}$ and a pair of slits at $x = \pm\frac{3d}{2}$. Write down $\mathcal{R}(\psi_x)$ as a quantity proportional to the modulus^2 of the sum of the diffracted amplitudes from the two pairs of slits.
Hint:
The standard result for a pair of thin slits of full separation D (Young's experiment) is

$$U(\psi_x) \propto \cos\left(\tfrac{kD}{2}\sin\psi_x\right) .$$

(c.)
Consider the aperture function for these four slits to be the convolution of a pair of δ-functions separated by d and another pair of δ-functions separated by $2d$ (both pairs are symmetric about $x = 0$). Write down $\mathcal{R}(\psi_x)$ as the product of two two-slit $\mathcal{R}$'s.
(d.)
Are your answers to parts (a.), (b.), and (c.) equivalent? Why or why not?
Hint:
Do all three methods represent valid approaches to the same physical problem?

71. Fuzzy thick slit.
Please use the notation and results of the previous problem. Consider a trapezoidal aperture function

$$\begin{aligned} g(x) &= 1 \qquad |x| < \tfrac{a}{2} \\ &= 0 \qquad |x| > a \\ &= \tfrac{2}{a}(x+a) \qquad -a < x < -\tfrac{a}{2} \\ &= \tfrac{2}{a}(a-x) \qquad \tfrac{a}{2} < x < a\ . \end{aligned}$$

Fraunhofer conditions apply. Under these conditions, calculate the slit's diffraction pattern

$$\mathcal{R}(\psi_x) \equiv \frac{I(\psi_x)}{I(\psi_x = 0)} .$$

Hint:
The convolution of two identical slits of thickness $\frac{D}{2}$ is an isosceles triangle of base D. This could be considered to be a trapezoid with a plateau of zero length. How would you adjust the two slit thicknesses to obtain the trapezoid base and plateau which the problem specifies?

72. Thick slits with wave plates.
A linearly ($\hat{x}$) polarized plane EM wave traveling along $\hat{z}$ is incident on an opaque baffle located in the plane $z = 0$. The baffle has two slits cut in it, which are of infinite extent in the $\hat{y}$ direction. In the $\hat{x}$ direction, the slit widths are each a and their center-to-center distance is d. (Obviously $d > a$, but you may *not* assume that $d \gg a$.) The top and bottom slits are each an equal distance from $x = 0$.

The diffracted image is viewed on a screen located in the plane $z = L$, where $L \gg d$; also $\lambda L \gg d^2$, where λ is the EM wavelength.

Quarter-wave plates are placed in each slit. They are identical, except that the top plate's "slow" (high-index) axis is along $(\hat{x}+\hat{y})/\sqrt{2}$ (+45° with respect to the $\hat{x}$ axis), while the bottom plate's slow axis is along $(\hat{x}-\hat{y})/\sqrt{2}$ (−45° with respect to the $\hat{x}$ axis).

(a.)

What is the state of polarization of the diffracted light that hits the center of the screen, at $x = y = 0$? Explain.

Hint:

Divide each thick slit into N contiguous thin slits, where $N \to \infty$. Out of N, consider the n^{th}-above-the-origin together with the n^{th}-below-the-origin thin slits as sources of diffracted light. When observed at the symmetry point $x = 0$, the optical paths from these two thin slits are the same. Therefore the polarization of the light from these two thin slits will be given by the sum of the Jones vectors for the light emerging from each slit.

(b.)

At what diffracted angle ψ_x does the first minimum of the irradiance occur?

Hint:

Does the light from the top slit interfere with the light from the bottom slit? See the result of problem 52.